

Possible Effects of Spacetime Foam in Particle Physics

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Abstract

We present an extension of quantum field theory to the case when spacetime topology fluctuates (spacetime foam). In this extension the number of bosonic fields becomes a variable and the ground state is characterized by a finite particle number density. It is shown that when the number of fields remains a constant, the standard field theory is restored. However, in the complete theory the ground state has a nontrivial properties. In particular, it produces an increase in the level of quantum fluctuations in the field potentials and an additional renormalization of masses of particles. We examine fluctuations of massless fields and show that in the presence of a temperature (thermal state) these fluctuations has $1/f$ spectrum. In the case of electromagnetic field this should produce an additional $1/f$ - noise in electric circuits.

1 Introduction

As was first pointed out by Wheeler at Planck scales the topology of space experiences quantum fluctuations (the so-called spacetime foam) [1] and since then, many attempts had been done to include the spacetime topology in the number of dynamical characteristics, e.g., see Refs. [2, 3]. The existence of such fluctuations means that the standard classical picture, in which the spacetime represents a smooth manifold, has only an approximate character and is apparently valid after some averaging out. In this sense one may speak only about an effective manifold corresponding to the physical space. The need to include topology changes in quantum gravity can be easily illustrated by the analogy with relativistic particles. Indeed, in particle physics there exists a minimal uncertainty in measurements of the position of a particle $\Delta x_{\min} \sim 1/m$ (the Compton length). In the processes, when Δx_{\min} may be considered as a small value (in comparison with characteristic scales involved), the number of particles remains a constant, while in the opposite case it represents an extra dynamical variable. We note that the number of particles is the simplest topological characteristic of the system, which consist of a set of particles, and, therefore, the change of this number gives the simplest example of topology transformations. In quantum gravity there also exists a similar uncertainty $\Delta g_{\min} \sim L_{pl}^2/L^2$ (where L_{pl} is the Planck length and L is a characteristic size of the region in which the gravitational field is measured). Therefore, quantum gravity processes, in which Δg_{\min} cannot be considered as a negligible value, have unavoidably to involve topology changes.

In this paper we will use the axiomatic approach (suggested in Ref. [4]) to construct the extension of quantum field theory to the case when the topology of spacetime is a variable and to examine possible effects of the spacetime foam in particle physics. This approach is based on a few basic principles (or axioms) and results in a generalization of the method of second

quantization (we call it as a second quantization of distributed systems). The first axiom can be formulated as follows:

Axiom 1 *All possible topologies of the effective physical space have to be described in terms of the basic Minkowski space.*

This axiom is not arbitrary but comes from the analysis of the role of the measurement procedure in quantum theory, e.g., see, Ref. [5]. Indeed, in quantum mechanics the measurement device has to obey classical laws (with a sufficient degree of accuracy) and, therefore, it always introduces peculiar restrictions on possible ways to describe and classify quantum states. In particular, spatial relationships are enforced by measurement instruments and thus, the fundamental role must play a basic space which comes from the measuring device. In the modern physical picture the role of such a basic space plays the ordinary Minkowski space (at least at laboratory scales). On the contrary, the effective manifold, which can be related to the physical space, plays a secondary role and is not, in general, directly observable. Moreover, in the case of an arbitrary quantum state this manifold does not possess a definite structure at all. We note that analogous principle was first used in Ref. [6] (see also references therein).

From this axiom we derive the following important consequence. Since in the general case the topology of the effective space differs from that of the basic space, therefore, upon the transformation to the basic space all physical observables will be described by multi-valued functions of coordinates. We can say that physical observables may admit a set of images in the Minkowski space and a change of the topology corresponds to the respective change in the number of images.

The next axiom represents the so-called identity principle:

Axiom 2 *Images of the same physical observable should obey the identity principle.*

This axiom generalizes the principle of indistinguishability of particles to the case of fields and other possible observables and it results in the fact that images of an arbitrary observable should obey to a definite statistics (Bose or Fermi). To fix the choice of the statistics we shall use an additional axiom as follows:

Axiom 3 *The statistics should be unique for all types of observables.*

This axiom comes from the fact that the number of images of an arbitrary observable is determined by the topology of the effective Riemannian space. In the most general case one can construct a quantum theory in which the last axiom does not work. However, in this case the relation of the number of images of observables to the topology of the effective space breaks down.

It follows, from the last two axioms, that *the only acceptable choice is Fermi -Dirac statistics*, since otherwise we will be unable to include in this scheme fermions.

Now, using these basic principles we can construct an extension of quantum field theory which is based on a generalization of the second quantization scheme. Beforehand, we recall that at a fundamental level, the composition of matter is determined by a set of fields and their sources. The sources are point particles, which in the standard quantum theory behave like fermions. The second quantization of sources gives the standard description of fermions exactly the same as in particle physics (we recall that by assumption there is only Fermi statistics). Thus the extension pointed out above involves bosonic fields.

2 Second quantization of distributed systems

Let M be the basic manifold (Minkowski space), φ be an arbitrary field on M , and A be a set of possible readings of an elementary detector (the elementary system of quantum numbers). More detailed consideration of this section (and subsequent section) can be found in Ref. [4]. Elements of A are a set of numbers $\xi = (\eta, \zeta)$, where ζ describes the position in space M^* at which the elementary measurement takes place (in what follows M^* denotes either the space M or the mode space) and $\eta \in V$ denotes either the field amplitude or the number of particles corresponding to the field. The result of a complete measurement of the field φ is the map $\varphi : M^* \rightarrow A$, which in the standard theory can be represented by functions $\eta(\zeta)$ on M^* with values in V .

The extension of the quantum field theory appears if we introduce the real physical space M_{ph} , which is assumed to have an arbitrary topology, and consider physical fields as an extended map of the form $\varphi^* : M_{ph}^* \rightarrow A$. Thus, the result of a real measurement will be a set of functions $\{\eta_J(\zeta)\}$, where $J = (i, \sigma_i)$, $\zeta \in \sigma_i$, and $\sigma_i \subset M^*$ is the range of the function $\eta_J(\zeta)$. Note that in general the dimensionality of the pieces σ_j can differ from that of M^* . Formal classification of such states can be reached as follows.

Let us introduce a set of operators $C^+(\xi)$ and $C(\xi)$, the creation and annihilation operators for an individual element of the set A . For the sake of simplicity we assume that coordinates ξ take discrete values. The Fermi statistics implies that these operators should satisfy the anticommutation relations

$$\{C(\xi)C^+(\xi')\}_+ = \delta_{\xi\xi'}. \quad (1)$$

We define the vacuum state $|0\rangle$ by the relationship $C(\xi)|0\rangle = 0$ and build a Fock space F in which the basis consists of the vectors $(n = 1, 2, \dots)$

$$|\xi_1, \xi_2, \dots, \xi_n\rangle = \prod_{i=1}^n C^+(\xi_i)|0\rangle. \quad (2)$$

The vacuum state corresponds to complete absence of a field and hence of all observables associated with the field. The state $|\xi\rangle$ describes the field φ with only one degree of freedom. This can be either a field concentrated at a single point or a field containing only one mode. States described by single-valued functions are constructed as follows:

$$|\eta(\zeta)\rangle = \prod_{\zeta \in M^*} C^+(\eta(\zeta), \zeta)|0\rangle. \quad (3)$$

Let us introduce the creation operator for a complex of degrees of freedom

$$D^+(\eta(\zeta), \sigma) = \prod_{\zeta \in \sigma} C^+(\eta(\zeta), \zeta), \quad (4)$$

where the domain of the function $\eta(\zeta)$ is limited to the set $\sigma \in M^*$. Then the states with an arbitrary number of fields can be written

$$|\eta_1, \eta_2, \dots, \eta_n\rangle = \prod_{i=1}^n D^+(\eta_i(\zeta), \sigma_i)|0\rangle. \quad (5)$$

3 Example of a scalar field

Consider as an example a real scalar field φ and for the sake of convenience we put the field in a box with length L and impose periodic boundary conditions. Consider the expansion of the field φ in plane waves,

$$\varphi(x) = \sum_k \left(2\omega_k L^3\right)^{-1/2} \left(a_k e^{ikx} + a_k^+ e^{-ikx}\right), \quad (6)$$

where $\omega_k = \sqrt{k^2 + m^2}$, and $k = 2\pi n/L$, with $n = (n_x, n_y, n_z)$. The general expression for the Hamiltonian is

$$H = H_0 + V(\varphi) \quad (7)$$

where H_0 describes free particles

$$H_0 = \sum_k \omega_k a_k^+ a_k \quad (8)$$

and the potential term V is responsible for the interaction. In the case of the scalar field the set A represents pairs $\xi = (a, k)$. The generalized second quantization results in the fact that the number of modes for every k becomes a variable and, therefore the set of variables $\{a_k, a_k^+\}$ is replaced by the somewhat expanded set $\{a_k(j), a_k^+(j)\}$, where $j \in [1, \dots, N_k]$, and N_k is the number of modes for a given wave number k . For a free field the energy is an additive quantity, which can be written

$$H_0 = \sum_k \sum_{j=1}^{N_k} \omega_k a_k^+(j) a_k(j). \quad (9)$$

Since the modes are indistinguishable, the interaction operator can be presented in the form (e.g., see Ref. [4])

$$V = \sum_{n,m,m'} \sum_{k_1, \dots, k_n}^l \sum_{j_1, \dots, j_n} V_{\{m\}, \{m'\}}^n(k_1, \dots, k_n) \prod_{i=1}^n \left(a_{k_i}^+(j_i)\right)^{m_i} \left(a_{k_i}(j_i)\right)^{m'_i}, \quad (10)$$

where the indices j_i run through the corresponding intervals $j_i \in [1, \dots, N(k_i)]$ and we assume that the sum with respect to the wave vectors k_i contains no terms with equal indices, i.e., $k_i \neq k_j$ for any pair of indices i and j .

The definition of V contains an ambiguity which corresponds to the ambiguity in possible ways to generalize terms of the type $(a)^m$, where $m > 1$, e.g., one can use one of the expressions

$$(a_k)^m \rightarrow \begin{cases} \sum_{j=1}^N (a_k(j))^m, \\ \sum_{j_1, j_2} (a_k(j_1))^{m-1} a_k(j_2), \\ \dots \\ \sum_{j_1, \dots, j_m} a_k(j_1) a_k(j_2) \dots a_k(j_m), \end{cases} \quad (11)$$

or any their combination with the demand that in the classical theory (when $N_k = 1$ for every k) we should obtain the correct classical expressions. It is remarkable, that this ambiguity reflects the renormalization procedure in the conventional quantum field theory and may provide a more deep physical ground for the renormalization theory.

Consider now the representation of occupation numbers. First, we introduce the notation

$$A_{m,n}(k) = \sum_{j=1}^{N(k)} \left(a_k^+(j)\right)^m \left(a_k(j)\right)^n. \quad (12)$$

Then the expression for the field Hamiltonian takes the form

$$H = \sum_k \omega_k A_{1,1}(k) + \sum_{n, \{m\}, \{m'\}} \sum'_{k_1, \dots, k_n} V_{\{m\}, \{m'\}}(k_1, \dots, k_n) \prod_{i=1}^n A_{m_i, m'_i}(k_i). \quad (13)$$

The fundamental operators $C^+(\xi)$ and $C(\xi)$ have the representation

$$C(a^*, k) = \sum_{n=0}^{\infty} C(n, k) \frac{(a^*)^n}{\sqrt{n!}}, \quad C^+(a, k) = \sum_{n=0}^{\infty} C^+(n, k) \frac{a^n}{\sqrt{n!}}, \quad (14)$$

where $C(n, k)$ and $C^+(n, k)$ obey the anticommutation relations

$$\{C(n, k), C^+(m, k')\} = \delta_{n,m} \delta_{k,k'}. \quad (15)$$

The physical meaning of the operators $C(n, k)$ and $C^+(n, k)$ is that they create and annihilate modes with a given number of particles. In terms of these operators we get

$$\hat{A}_{m_1, m_2}(k) = \sum_{n=0}^{\infty} \frac{\sqrt{(n+m_1)!(n+m_2)!}}{n!} C^+(n+m_1, k) C(n+m_2, k) \quad (16)$$

and expression for the Hamiltonian in terms of the operators $C^+(\xi)$ and $C(\xi)$ can be obtained by substituting (16) into (13). For a free field, the eigenvalues of the Hamiltonian take the form

$$\hat{H}_0 = \sum_k \omega_k \hat{A}_{1,1}(k) = \sum_{n,k} n \omega_k N_{n,k}, \quad (17)$$

where $N_{n,k}$ is the number of modes for fixed values of the wave number k and the number of particles n ($N_{n,k} = C^+(n, k) C(n, k)$).

Thus, the field state vector Φ is a function of the occupation numbers $\Phi(N_{k,n}, t)$, and its evolution is described by the Schrodinger equation

$$i\partial_t \Phi = H\Phi. \quad (18)$$

4 Physical particles and effective field

Consider now the problem of representing physical observable particles. Among the operators (16) we distinguish some that change the number of particles by one:

$$b_m(k) = \hat{A}_{m, m+1}(k), \quad b_m^+(k) = \hat{A}_{m+1, m}(k), \quad (19)$$

$$[\hat{n}, b_m^{(+)}(k)] = \pm b_m^{(+)}(k), \quad [H_0, b_m^{(+)}(k)] = \pm \omega_k b_m^{(+)}(k), \quad (20)$$

where

$$\hat{n} = \sum_k \hat{n}_k = \sum_{n,k} n N_{n,k}. \quad (21)$$

Then we can define the ground state Φ_0 (where $b_m(k) \Phi_0 = 0$ for arbitrary $m = 0, 1, \dots$ and k) which corresponds to the minimum energy for a fixed mode distribution N_k . Note that in contrast to the standard theory, the ground state is generally characterized by a nonvanishing particle number density $\hat{n}\Phi_0 = n_0\Phi_0$ and is not a Lorentz invariant state.

In the absence of processes related to changes in the topology of space and for a mode distribution of the form $N_k = 1$ (there is only one mode for each wave number k), the standard

field theory is restored. Furthermore, there is a rather general case in which the concept of an effective field can be introduced to restore the standard picture.

Indeed, consider the case in which the interaction operator in (13) is expressed solely in terms of the set of operators $b_0(k)$ and $b_0^+(k)$ and the operator $N_k = \sum_n C^+(n, k) C(n, k)$ is an integral of motion (i.e., $[N_k, H] = 0$). Then we can renormalize the set of operators $b_0(k)$ and $b_0^+(k)$ as follows

$$d_k = N_k^{-1/2} b_0(k), \quad d_k^+ = N_k^{-1/2} b_0^+(k), \quad (22)$$

and restore the standard commutation relations (this follows from (15) and (16))

$$[d_k, d_{k'}^+] = \delta_{kk'}. \quad (23)$$

Thus, the standard algebra is restored and the set of operators $\{d_k, d_k^+\}$ can be used to define an effective field $\tilde{\varphi}$ whose quanta coincide with physical particles in the complete (extended) theory. One may consider the effective field as a renormalized field φ . The basis of the Fock space will consist of the vectors $\{d_{k_1}^+ d_{k_2}^+ \dots d_{k_n}^+ \Phi_0\}$. The ground state Φ_0 is, in general, not a Lorentz invariant state, for it contains a nonvanishing particle number and energy densities: $\hat{n}_k \Phi_0 = \bar{n}_k \Phi_0$ and $H_{0k} \Phi_0 = e_k \Phi_0$, where $e_k = \bar{n}_k \omega_k$ and \hat{n}_k is the operator defined in (21). We stress that this does not mean that the Lorentz invariance of the theory is broken. This situation is completely analogous to the spontaneous break of symmetry in gauge theories. The physical particles are excitations of the ground state and for the number of particles operator we get $d_k^+ d_k = \delta \hat{n}_k = \hat{n}_k - \bar{n}_k$. However, the properties of the ground state Φ_0 remain beyond the scope of the effective field and should be determined in the complete theory.

5 Ground state and possible effects

Consider now properties of the ground state Φ_0 . First, we note that there exists a trivial ground state which coincides with the vacuum state of the ordinary quantum field theory (in which $N_k = 1$ for all k). In this case our theory reduces to the standard one. However, we recall that in the very beginning of the evolution our Universe has passed the quantum stage. During this period processes involving topology changes take place and this should result in some excess of the field degrees of freedom (modes). After the quantum stage, processes with topology changes are suppressed (e.g., see experimental restrictions in Refs. [7]), and the structure of space is preserved. Thus, we may expect that in the modern universe the ground state has a nontrivial structure. In the simplest case the field ground state Φ_0 is characterized by occupation numbers of the type

$$N_{k,n} = \theta(\mu - n\omega_k), \quad (24)$$

where $\theta(x)$ is the Heaviside step function and μ is the chemical potential. Thus, for the mode spectral density we get

$$N_k = \sum_{n=0}^{\infty} \theta(\mu - n\omega_k) = 1 + \left[\frac{\mu}{\omega_k} \right], \quad (25)$$

where $[x]$ denotes the integer part of the number x . Equation (25) shows that at $\omega_k > \mu$ we have $N_k = 1$, i.e., the field structure corresponds to a flat Minkowski space, while in the range $\omega_k < \mu$ we have $N_k > 1$ and the field should exhibit a nontrivial properties. In particular, this state is characterized by the spectral density of the number of particles

$$\bar{n}_k = \sum_{n=0}^{\infty} n \theta(\mu - n\omega_k) = \frac{1}{2} \left(1 + \left[\frac{\mu}{\omega_k} \right] \right) \left[\frac{\mu}{\omega_k} \right] \quad (26)$$

and the spectral density of the ground-state energy

$$e_k = \omega_k \bar{n}_k = \frac{\omega_k}{2} \left(1 + \left[\frac{\mu}{\omega_k} \right] \right) \left[\frac{\mu}{\omega_k} \right]. \quad (27)$$

Since the given particles correspond to the ground state of the field, in ordinary processes (which do not involve topology changes) these particles are not manifested explicitly. From the effective-field standpoint, such particles are "dark" and should comprise dark matter of the universe (properties of this dark matter were considered in Ref.[4]). In what follows we discuss one main effect which admit experimental verification. It is an increase in the level of fluctuations in field potentials. In the case of massless particles field potentials are measurable quantities and are of special interest.

Consider fluctuations of the field potentials in the ground state (24). From (6), (16), and (24) we find

$$\langle \varphi(x) \varphi(x+r) \rangle = \frac{1}{(2\pi)^2} \int_0^\infty \frac{dk}{\omega_k} \frac{\sin kr}{kr} \Phi^2(k), \quad (28)$$

where

$$\Phi^2(k) = k^2 N_k = k^2 \left(1 + \left[\frac{\mu}{\omega_k} \right] \right).$$

Thus, the presence of the "dark" particles in the ground state provides the increase in the level of quantum fluctuations of field potentials $\delta\Phi^2 = k^2 \left[\frac{\mu}{\omega_k} \right]$ in comparison with the standard vacuum noise ($\mu = 0$). In the case of massless fields this increase is substantial at long wavelengths $k \ll \mu$, where $\delta\Phi^2 \simeq k\mu$.

In the present universe fields are characterized by a nonvanishing temperature T . Thus, instead of the ground state we should use the thermal equilibrium state which is characterized by occupation numbers of the type $N_{k,n} = \left(\exp \left(\frac{\mu - n\omega_k}{T} \right) + 1 \right)^{-1}$. Since processes associated with topology changes are the first to stop during the early stages of the evolution of the universe, one should expect that $T \ll T_\gamma$ (T_γ is temperature of the microwave background radiation). On the other hand, for the value of μ one can obtain an upper bound $\mu^* \sim 10^2 T_\gamma$ (which corresponds to the situation when all observable dark matter is formed by the hidden particles). Then for massless fields we find that at long wavelengths $k \ll T \ll \mu$ field fluctuations turn out to be scale-independent $\delta\Phi^2 \simeq T\mu$. In the case of the electromagnetic field such fluctuations can be easily observed. Indeed, in such a field any conductive sample will exhibit current fluctuations with spectrum $1/f$. We note that unlike the standard $1/f$ -noise these fluctuations have the external origin and, therefore, have a universal nature. In particular they do exist in the absence of a constant current in the sample.

In conclusion we consider a renormalization of masses of physical particles which takes place in case of fields with self-action and caused by the presence of the hidden particles in the ground state. Consider as an example the real scalar field with the potential in (7) of the type $V = \frac{\lambda}{4!} \varphi^4$. Then the observable value of the mass of particles will be $m_{ph}^2 = m_0^2 + \frac{\lambda}{2} \langle \varphi^2 \rangle$, where m_0^2 is the initial value (for $\mu = 0$) and we assume $\lambda \ll 1$. Thus we find

$$\delta m^2 = \begin{cases} \frac{\lambda}{(4\pi)^2} \xi(2) \mu^2 & \text{as } m_0 = 0, \\ \frac{\lambda}{24\pi^2} \frac{z^3}{m_0} & \text{as } m_0 \neq 0, \end{cases} \quad (29)$$

where $z^2 = \mu^2 - m_0^2$, $\xi(2) = \sum_{n=1}^\infty \frac{1}{n^2}$, and we assume $z \ll m_0$.

6 Summary

In this manner the extension of the field theory proposed predicts a number of new effects which in principle admit experimental verification. First of all the ground state of the theory, which represents the vacuum state from the standard field theory standpoint, is characterized by a finite density of particles. These particles are dark and contribute to the dark matter of the universe. The presence of particles in the ground state results in the fact that this state is not a Lorentz invariant state, though the complete theory remains to be invariant one. This situation is similar to the spontaneous symmetry breaking in gauge theories.

The presence of the dark particles leads also to an additional renormalization of parameters of physical particles and an increase in the level of quantum fluctuations in field potentials. The last effect is crucial for the experimental verification of the theory proposed. Indeed, for massless fields the increase is essential at small wavelengths and in the presence of a temperature field fluctuations are scale-independent. This means that the present universe is filled with a random electromagnetic field which at scales $k \ll T_\gamma$ (T_γ is the temperature of the microwave background radiation) has $1/f$ spectrum $\delta E^2 \simeq T_\gamma \mu \leq 10^2 T_\gamma^2$. In any electric circuit this field should cause an additional additive $1/f$ -noise, which due to the external origin should be correlated in different circuits. Direct observations of such a noise will allow to determine the value or limits of the parameter μ which characterizes the nontrivial topological structure of space.

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